Lecture 30 (11/12/2021)

* Prayer
* Spiritual thought

Laplace transform's properties:

- Linearity: $\mathcal{L}\{f+g\} = \mathcal{L}\{f\} + \mathcal{L}\{g\}$

  $\mathcal{L}\{cf\} = c\mathcal{L}\{f\}$

  $\mathcal{L}\{fg\} \neq \mathcal{L}\{f\}\mathcal{L}\{g\}$
- Connection with derivative:

  $\mathcal{L}\{f'\} = s\mathcal{L}\{f\} - f(0)$
- Connection with convolution:

  $\mathcal{L}\{f*g\} = \mathcal{L}\{f\}\mathcal{L}\{g\}$

Definition of convolution:

$$f*g(t) = \int_0^t f(s)g(t-s)\,ds$$

Application in image processing (filtering) and in regularity theory of PDEs.

Ex:
$$\mathcal{L}^{-1}\left\{\frac{s^2}{(s^2+2s+5)^2}\right\}$$

$$F(s) = \frac{s^2}{(s^2+2s+5)^2} = \underbrace{\frac{s}{s^2+2s+5}}_{G(s)} \cdot \underbrace{\frac{s}{s^2+2s+5}}_{G(s)}$$

$f = \mathcal{L}^{-1}\{F\}$

$g = \mathcal{L}^{-1}\{G\} = e^{-t}\left(\cos 2t - \frac{1}{2}\sin 2t\right)$

$$\rightsquigarrow f(t) = g*g(t) = e^{-t}\int_0^t \left(\cos 2s - \frac{1}{2}\sin 2s\right)\left(\cos(2t-2s) - \frac{1}{2}\sin(2t-2s)\right)ds$$

$$= \cdots \text{ (use trig-identities)}$$